

Learning Math as an Experimental Science

Art Bardige sustainablearning.org

In the year **1988**



The Hubble Space Telescope starts operating

Mathematics became "The Science of Patterns"



Professor Lynn Steen, 1988

The rapid growth of computing and applications has helped cross-fertilize the mathematical sciences, yielding an unprecedented abundance of new methods, theories, and models. Examples from statistical science, core mathematics, and applied mathematics illustrate these changes, which have both broadened and enriched the relation between mathematics and science. No longer just the study of number and space, mathematical science has become the science of patterns, with theory built on relations among patterns and on applications derived from the fit between pattern and observation.

What if...

students could learn mathematics as an experimental science?





Using spreadsheets as laboratories



To solve interesting Problems that "instill a passion" for learning, by:



Enrico Fermi

Building models



2010 Winter Olympics Medal Standings

Nation 💌	Gold 💌	Silver 🔻	Bronze 💌	Total 💌	1924-2010	▼ Per Participant ▼
United States	9	15	13	37		<u>ر 0.17</u>
Germany	10	13	7	30	~~~~~	0.20
Canada	14	7	5	26		0.13
Norway	9	8	6	23	~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~	···· 0.23
Austria	4	6	6	16	/	
Russia	3	5	7	15		/* 0.08
South Korea	6	6	2	14		~^ 0.30
China	5	2	4	11		0.10
France	2	3	6	11	$\sim\sim$	0.10
Sweden	5	2	4	11	~~~~	./ 0.12
Switzerland	6	0	3	9	^	0.06
Netherlands	4	1	3	8		<u>^</u>
Czech Republic	2	0	4	6		.007
Poland	1	3	2	6		
Japan	0	3	2	5		1.05
Italy	1	1	3	5		0.05
Finland	0	1	4	5	un.	·~ 0.05
Slovenia	0	2	1	3		0.04
Slovakia	1	1	1	3		
Croatia	0	2	1	3		
Australia	2	1	0	3		
Belarus	1	1	1	3		∕√ 0.16
Latvia	0	2	0	2		
Kazakhstan	0	1	0	1		1.02 €
Estonia	0	1	0	1		_ [*] 0.03
Great Britain	1	0	0	1	$\sim\sim\sim$	~ 0.03
Use the dropdown arrows next to each of the column headings to sort or filter the medal standings.						

Analyzing data

Visualizing results



And asking

what if... เกิดอะไรขึ้นถ้า ماذا لو 만약 如果 Hvad nu hvis Que faire si Cosa succede se Was ist, wenn ¿Qué pasa si quod si Τι θα συμβεί αν どのような場合 מה היה קורה אילו Что делать, если E si Hvað ef

Before we look into the future let's see from whence we came.

In the year **1202**



King Richard, The Lionhearted died, 1199

A revolutionary book was published



Virtually unknown today



"The Book of Calculation"



By Leonardo of Pisa



aka "Fibonacci"



Leonardo Di Vinci

It had a greater impact than the works of the other Leonardo

Born in Pisa





He grew up in the 1170's along with the Leaning Tower



When Pisa was a great trading city

Merchants and traders used Roman math and an abacus to calculate



But multiplying by			
doubling was			
slow, cumbersome,			
and error prone			

CLXVII	Ι
CLXVIICLXVII	II
CLXVIICLXVIICLXVII CLXVII	IV
CLXVIICLXVIICLXVII CLXVIICLXVIICLXVII CLXVIICLXVII	VIII
CLXVIICLXVIICLXVII CLXVIICLXVIICLXVII CLXVIICLXVIICLXVII	I+VIII=IX

167 x 9 =

It worked for the Empire

None of the cities should be allowed to have its own separate coinage or a system of weights and measures; they should all be required to use ours.

Dio Cassius

But not for medieval trade when city-states had



Different weights, measures, and currency



Leonardo joined his father, a trader, and customs official in Algeria



Where he learned Arabic arithmetic and algebra, become a trader and returned to Pisa in 1200

The Compendious Book on Calculation by Completion and Balancing by al Khwarizmi

Enabled by the new technology of paper to track algorithmic calculations



Paper was introduced into Europe around 1100.

Leonardo developed a revolutionary mathematics for business.

The Chapters in Liber abbaci (1202)

- 1. On the recognition of the nine Indian figures and how all numbers are written with them.
- 2. On the multiplication of whole numbers
- 3. On the addition of them, one to another
- 4. On the subtraction of lesser numbers from greater numbers
- 5. On the division of integral numbers
- 6. On the multiplication of integral numbers with fractions
- 7. On the addition and subtraction and division of numbers and fractions and the reduction of parts to a single part
- 8. On the buying and selling of commercial things *(ratio & proportion)*
- 9. On the barter of commercial things (rate)
- 10. On companies made among parties (percents)
- 11. On the alloying of money (mixture problems)
- 12. On the solutions of many problems (Fibonacci sequence)
- 13. On the rule of elchataym by which problems of false position are solved. *(solving linear equations)*
- 14. On the finding of square and cube roots, on binomials and their roots.
- 15. On the pertinent rules of geometric proportions

Leonardo's algorithmic math "arithmetic necessary to merchants" Fully symbolic by 1600...



Algorist vs. abacist (1508)

...continues to be the basal arithmetic and algebra every student is required to master.

COMMON CORE STATE STANDARDS FOR



In the year **1979**



A technology revolution changed the mathematics of business



Dan Bricklin, a frustrated



Bob Frankston & Dan Bricklin

Harvard Business School student



Working case studies



Found calculators slow, cumbersome, and error prone



So he invented the spreadsheet

120 (U) +H20	3*12		ĥ
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	NOV. 2500.00	DEC. 2500.00	TOTAL 30000.00
INCOME	2500.00	2500.00	30000.00
FOOD PRENT 10 REAT 10 REC. 10 REC. 10 REC. 10 REC. 10 REC. 10 REC. 10 REAT 10 REC. 10 REAT 10 REC. 10 REAT 10 REAT	400.00 350.00 100.00 1000.00 1000.00 1000.00 300.00	400.00 350.00 120.00 100.00 1000.00 1000.00 100.00 300.00	4800.00 4200.00 575.00 1200.00 1200.00 1200.00 1200.00 3600.00
17 EXPENSES	2460.00	2470.00	28775.00
19REMAINDER 20SAVINGS	40.00 30.00	30.00 30.00	1225.00 360.00

VisiCalc the Visible Calculator (1979)

Mitch Kapor added graphs



VisiPlot 1980

And then a database



Mitch and Lotus 123 in 1982

Putting a PC on every business desk and...



The IBM PC

A spreadsheet on every computer



1995

Spreadsheets are function machines



Function is a fundamental concept



HARVARD COLLEGE Handbook for Students

Mathematics Professor Peter Kronheimer, Director of Undergraduate Studies

Mathematics is the science of order, and mathematicians seek to identify instances of order and to formulate and understand concepts that enable us to perceive order in complicated situations.

Perhaps the most important concept of mathematics is that of function, which provides us with the means to study dependence and change.

Quarterly Cash Flow Projection

The ABC Corporation First Quarter, 2010

CEDDIIADV

Variance (\$6,515 (2,649 (1,272 (6,780

(\$17,216

(354 (219 735 1,022

467 (1,424 1,467 (5,010

(448) 47 (1,364) (157) 384 (\$5,623) (\$1,347) 240 (5,000) (\$11,730) (\$5,486)

Enables business to solve dynamic problems

	JANUART			FEDRUART		
Cash Received	Estimate	Actual	Variance	Estimate	Actual	
Beginning Cash Balance	\$52,552	\$54,530	(\$1,978)	\$36,247	\$42,762	
Cash Sales	23,821	25,485	(1,664)	23,819	26,468	
Collections	59,014	61,001	(1,987)	62,394	63,666	
Loans	15,121	18,452	(3,331)	12,329	19,109	
Total Cash Available	\$150,508	\$159,468	(\$8,960)	\$134,789	\$152,005	
Cash Disbursed						
Salaries and Wages	\$14,752	\$15,855	(\$1,103)	\$12,890	\$13,659	
Lease/Mortgage	1,659	1,659	0	1,422	1,776	
Insurance	1,041	1,085	(44)	956	1,175	
Office Supplies	5,404	3,505	1,899	4,646	3,911	
Utilities	7,316	7,105	211	8,212	7,190	
Repairs and Maintenance	4,378	4,155	223	4,161	3,694	
Operating Supplies	2,805	3,950	(1,145)	3,114	4,538	
Professional Fees	3,437	1,575	1,862	3,097	1,630	
Commissions	13,716	15,850	(2,134)	12,293	17,303	
Travel and Entertainment	3,734	4,580	(846)	4,061	4,509	
Purchases	4,140	4,500	(360)	4,830	4,783	
Advertising	1,957	2,850	(893)	1,841	3,205	
Transportation	14,776	13,505	1,271	12,681	12,838	
Other	12,352	11,318	1,034	12,794	12,410	
Total Disbursements	\$91,467	\$91,492	(\$25)	\$86,998	\$92,621	
Cash Position						
Loan Payment with Interest	\$8,214	\$8,214	\$0	\$7,318	\$8,665	
Capital Purchases	2,080	1,500	580	2,004	1,764	
Owner's Withdrawal	12,500	15,500	(3,000)	5,000	10,000	
Total Cash Paid Out	\$114,261	\$116,706	(\$2,445)	\$101,320	\$113,050	
End Of Month	\$36,247	\$42,762	(\$6,515)	\$33,469	\$38,955	

LA MILLA DO

MARCH					
Estimate	Actual	Variance			
\$33,469	\$38,955	(\$5,486			
25,443	26,780	(1,337			
73,702	69,487	4,215			
14,513	19,452	(4,939			
\$147,127	\$154,674	\$7,547			

\$15,042	\$14,150	\$892
1,589	1,778	(189)
980	1,328	(348)
5,395	4,676	719
6,837	7,535	(698)
4,766	4,330	436
3,247	5,170	(1,923)
3,245	1,514	1,731
11,060	19,577	(8,517)
4,546	4,923	(377)
4,161	5,129	(968)
1,700	3,499	(1,799)
10,522	14,371	(3,849)
10,630	13,920	(3,290)
\$83,720	\$101,900	(\$18,180)

\$6,353	\$8,278	(\$1,925)
2,348	1,493	855
15,000	10,000	5,000
\$107,421	\$121,671	(\$14,250)
\$39,706	\$33,003	\$6,703

We no longer use Leonardo's math to solve equations or to calculate on paper.



TK!Solver by Bricklin and Frankston 1982, long forgotten



Leonardo's algorithmically defined scope and sequence is unnecessary.

Mathematics S	Scope & Sequence	Kindergarten	1st Grade	2nd Grade	3rd Grade	4th Grade	5th Grade	6th Grade
Arithmetic								
Numbers	Wholes	objects, tallies, #0-20	1, 2, 10's even/odd	15 to 100, 1st-10th	610, ordinals, squares	powers of 10	powers of 2, primes	roots
	Place Value			(10)'s, (100)'s, (1/10)'s	(1000)'s, (.01)'s	.001 to 1,000,000	expanded notation, %	exponent/sci. notation
	Bases				2, 3, 4, 5	2, 8	16	formula for changing bases
	Unknowns		single	substitute 1, 2	in every location	changing unknowns	ratio problems	in motion problems
	Integers					negative numbers	+/- by a postive #	
	Rationals		parts of a whole	simple fractions	compare, order 1/D	compare/order N/D	equivalent, mixed #'s	conversions - fraction-dec
	Vectors		postion on a numberline	interval on # line	direction on a numberline	direction on a numberline	+-* by a constant	motion
Expressions	Evaluating	1 to 1 correspondence		single ()	multiple ()	embedded (), siimplifying	evaluate	Boolean logic
	Grouping	sort objects	group by 2, 10's	group/partition	regrouping, arrays	factors & multiples	order of operations	properties of arithmetic
	Operations	compare/order sets	+, - objects	+, - facts to 20	+, * facts, +, - same denom	facts to 12's, +, - fractions	fraction-whole: powers	%; =- integers
Relations	Algorithms	counting on	+, - columns	+, - (2 places)	+, -, regrouping	*, /	+-*/ decimals	alternative algorithms
	Equations	equalize groups	<, >, =, +, -	+, - open sentences	+, -, sentences to 100	*,/ sentences	ratios	numerical equations
	Functions			oriented operations	uinary operations	*, / by 10's, 100's	formulas	functional notation
Statistics								
	Probability			intuitive	chance	as a ratio	random sample; decimal	Pascal's Triangle
	Statistics	collect data	collect data	graphical average	numerical average	Mean	Standard Deviation	Mode, Median, Range
	Representation	picture graph	bar graph			line graph	area under a graph	"Bell Curves"
Geometry								
Shapes	Shapes	describe	2 dimensional families	classify	regular, irregular	rays, segments, solids	polygons, polyhedra	circles
	Symmetry	relate to objects		symmetry/congruency	lines of symmetry	parallel, perpendicular	construction of triangles	similarity, scale drawings
Relations	Angles	patterns	parts of a shape		compare angles	right angles	construct angles	rules of symmetry
	Perimeter	inside/outside		measuring	regular shapes	inside/ouside angle	sum of angles in a polygon	
Metric	Area		closed paths	counting squares	compare/order	regular polygons	polygons; formulas	circumference, p
	Volume		covering space			squares, rectangles	polygons	formulas, circles
Mapping	Direction	location	order/connect points	N, S, E, W	units of distance	self-derived formulas	polyhedra	spheres
	Position	above, below	left, right, top, bottom	label points	N, S, E, W (globe, map)	locating points grid		
Graphing		picture, bar	labeled bar	grid locations	collect data	ordered pairs		
Measurement	t			-	reading graphs	line		
Time	Events, Durations	before, during, after	order durations	order many events	duration between events	1/10's, 1/100's		
	Clocks	day, night	hour/half	5 minutes	minutes	digital displays		
	Calendar	days, seasons	order days, name months	order months, seasons	calendars	century, decades		
Objects & Space	e Distance	compare, order, equalize	compare	rulers	factions of an inch	miles, km		
	Volume	big, small, closer to	invented measures	scales, liquids	liter			

they were unanimously of opinion "that a radical change in the teaching of arithmetic was necessary. The Committee of Ten 1894



In the year 2013



We are launching What if Math

Practice problem solving not calculating

The MORE that you READ. the More THINGS you will KNOW. The MORE you LEARN. the More PLACES you'll GO! ~ Dr. Seuss

Perform Spreadsheet Experiments <u>Developed by</u> <u>students for</u> <u>students</u>



WIKIPEDIA The Free Encyclopedia



On a wide range of topics Integrating Science Technology Engineering Math



Financial Literacy



H The Problem Principal Rate Time Monthly Payments What If Lab Notes 2

Sustainability



How much electrical power would be saved if every home in the United States replaced one old incandescent bulb with the new CFL's.

Statistical Literacy



And even the arts



Based on Interesting Problems



Introduced with a good story

The Problem A Plan	what if math 👋 Napoleon's Pyramie
A Model	Texture in the second
Make a Table	During the Napoleonic campaign in
Find a Formula	Egypt in 1798, a famous battle was
A Wall	fought outside Cairo in Embaba, in
Was he Right?	Battle of the Pyramids, It is reported
What if	that, after the battle, Napoleon and
Lab Notes	his staff officers visited the Great Pyramid of Giza. While the more adventurous officers climbed to the top, Napoleon was content to rest in the shade of the pyramid at its base to wait for them to return.
	When the officers descended and rejoined him to brag about their physical strength, Napoleon announce that he had made a calculation of the amount of stone in the pyramid. There was enough, he said, to but a stone wall 3 meters high and 1/3 meter thick that would enclose the whole of France.
Was Napoleon right	Write and explain your conjecture here.

Following a consistent experimental method

1000	what if ma		Experimental Plan for	(Fill in the title)		
		Steps	Your Description			
1	1 Problem Formulate a question or a problem that would interest other students and tell a short story to capture their attention.		Write a description for each step in this	s column.		
2	Plan	Describe a sequence of steps to solve the problem. "What are we going to do in our spreadsheet lab?"				
3	3 Model Start by building a simple mathematical model for students to use, for example a simple table.					
4	Pattern	Extend the pattern. Add scope, data, or complexity to build a more complete model.				
5	Concept	Develop a explanation for the pattern, where it comes from, and how it illuminates a solution.				
6	What if	Suggest What if questions that lead students to explore the pattern more deeply.				
7	Reflect	Think of "What did I learn today?"questions for the Student Lab Notebook.				

Focused on patterns and patternmaking



Spread the word we want **Nerds**



Rich Twp. H.S. Rocket Club 1959

Share Interesting Problems



Hot Hands

What's Watt



Just TBuilt

Plastic Money



To create and share Interesting Problems

Pennies to Heaven

By a Hair



By a Hair



Could Ants be Human?



Snow White



Green Skin: Sugar High





Develop and Share great experiments



Win prizes building a library of thousands of great experiments enabling every student to...









Learn math as an experimental science



Whatifmath.org